On the Synthesis of Dual-Resonant Coaxial Cavities*

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Summary—Mathematical analyses of multisection coaxial cavities predict the possibility of shifting one of the spurious resonant frequencies (present in all coaxial cavities) to a desired frequency, thus allowing one cavity to do the work of two. The specific problem considered in this paper is the design of cavities to resonate a terminating capacitance (*e.g.*, tube capacitance, "varactor" capacitance) at two harmonically-related frequencies, with the additional requirement that the two frequencies remain very nearly in the desired ratio despite wide variations in the magnitude of the terminating capacitance. (As one obvious application, a cavity meeting these requirements would make possible an inherently-aligned single-cavity frequency multiplier.)

Curves based on computed results for specific cases are presented. Experimental cavities constructed according to the predicted designs have exhibited performance which is in very close agreement with the analysis, thus verifying both the validity of the method of analysis and the feasibility of the desired result.

Application of the same techniques to other and more general problems (*e.g.*, single-cavity mixers, voltage-tunable filters) are suggested.

I. Introduction

In the UHF frequency range, coaxial cavities are widely used as resonant elements because they provide excellent performance and lend themselves to straightforward design, production, and tuning methods. Disadvantages include the spurious resonant modes frequently encountered, and the excessive size, weight, and complexity of the multiple-cavity circuits needed for all except the simplest applications. Thus, the paradoxical situation exists that coaxial cavities have extra (and usually unwanted) resonant frequencies, but that in a typical application a number of cavities must be used in order to obtain resonance at the desired set of frequencies.

This paper considers the use of multiple-section cavities to shift a spurious resonant mode to a desired multiple of the fundamental frequency in such a way that the resonant frequency ratio is insensitive to variations in the terminating capacitance (*i.e.*, the desired frequency ratio is maintained within close limits in spite of wide variations in the terminating capacitance). Insensitivity to capacitance variation eliminates the need for selection or tuning of cavities to match individual tubes or allows the use of such cavities with time-varying capacitances.

Results of the analyses of three different cavity configurations are presented. These analyses, based on lossless approximations to the actual cavities, indicate that the desired results can be achieved with cavities of the types considered. Experimental results are presented which verify both the feasibility of the desired result and the validity of the analysis procedure. For purposes of comparison, uniform coaxial cavities are also investigated (Appendix I), and it is shown that the desired results can not be obtained with such cavities.

II. Analysis

A uniform lossless coaxial cavity of length L and characteristic impedance Z_0, short-circuited at one end and with a capacitance C connected across the other end, resonates at an infinite number of frequencies f each of which satisfies the following transcendental equation (neglecting end effects):

$$\frac{1}{j2\pi fC} + jZ_0 \tan 2\pi f \frac{L}{c} = 0 \qquad (1)$$

where c is the velocity of propagation of electromagnetic radiation in the line under consideration. Except in certain special cases (considered in Appendix I), these frequencies are not harmonically related.

In certain applications (*e.g.*, frequency multipliers, mixers) it is desirable to have a single cavity resonate a terminating capacitance (*e.g.*, tube capacitance, diode capacitance) at two harmonically-related frequencies. (Cavities having this property will be said to satisfy the dual-resonance condition.) Furthermore, the frequency ratio should be insensitive to variations in the terminating capacitance, so that the dual-resonance condition is maintained, to a good approximation, over a range of terminating capacitance variation. A cavity design which satisfied this condition (henceforth referred to as the insensitivity condition) would be particularly desirable in applications where the terminating capacitance changes with time or where production or cost requirements preclude the tuning or matching of particular cavities to particular tubes. It is shown below that coaxial cavities of the types depicted in Fig. 1 can be designed to produce the desired results, whereas uniform coaxial cavities can never meet both conditions (see Appendix I).

These two conditions will now be stated in terms of the cavity variables and parameters: let $Z(f)$ represent the impedance of the cavity, as seen from the open end,

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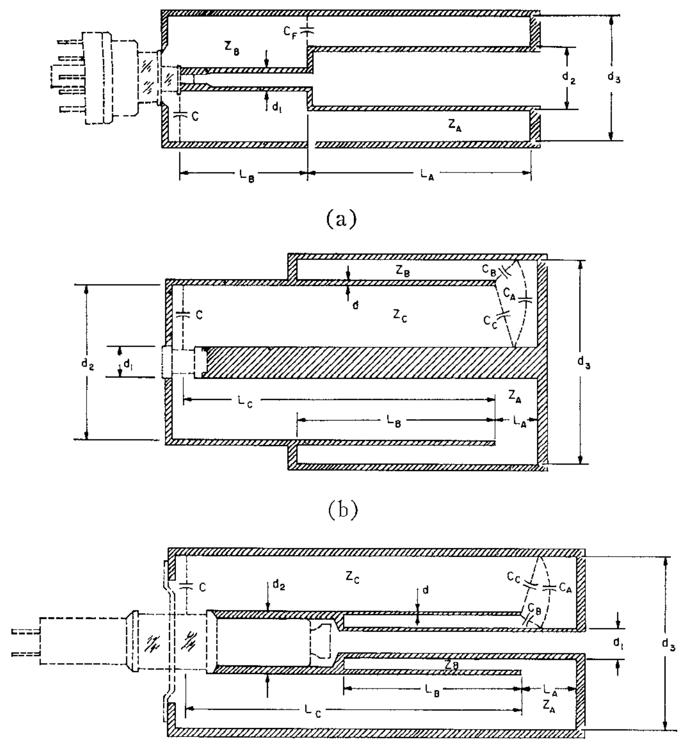


Fig. 1—Cavity configurations: (a) in-line, showing use with "lighthouse" triode, (b) external re-entrant, showing use with "varactor," (c) internal re-entrant, showing use with "pencil" triode. (C is the equivalent capacitance of the tube or "varactor.")

where $Z(f)$ is a given function[1] of f. For each value of terminating capacitance C, any frequency which satisfies the relation

$$Z(f) + \frac{1}{j2\pi fC} = 0 \qquad \text{(resonance condition)} \qquad (2)$$

is a resonant frequency. Assume that there exists a pair of solutions to (2), $f=f_1(C)$ and $f=f_2(C)$. Ideally, f_2 should equal nf_1 for all values of C, where n is the desired frequency ratio. This is not possible, in general, but can be approximated[2] over a limited range of C near C_0, if $Z(f)$ is chosen so that

$$f_2(C_0) = nf_1(C_0) \qquad \text{(dual-resonance condition)} \qquad (3)$$

and

$$\left.\frac{df_2(C)}{dC}\right|_{C_0} = n\left.\frac{df_1(C)}{dC}\right|_{C_0} \qquad \text{(insensitivity condition).} \qquad (4)$$

These conditions will now be rewritten in a more usable form: Substitution of (2) into (3) yields

$$Z[f_1(C_0)] = nZ[f_2(C_0)] \qquad \text{(dual-resonance condition)} \qquad (5)$$

[1] The detailed expressions used for each type of cavity are given by Waltz [1].

[2] This approximation makes use of the first two terms of the Taylor's-series expansions of $f_1(C)$ and $f_2(C)$ about $C=C_0$. A better approximation could be achieved by considering more terms in the series, at the cost of greatly increased computational difficulties.

as an equivalent to (3). Evaluation of df/dc from (2), (3) and (5) allows (4) to be rewritten as

$$\left.\frac{\partial Z(f)}{\partial f}\right|_{f=f_1(C_0)} = n^2 \left.\frac{\partial Z(f)}{\partial f}\right|_{f=nf_1(C_0)}$$

$$\text{(insensitivity condition).} \qquad (6)$$

Thus, the satisfaction of (2) at $f=f_1(C_0)$, and of (5) and (6), is sufficient to guarantee that $Z(f)$ satisfies both the dual-resonance condition and the insensitivity condition at $C=C_0$.

Analsis of In-Line Cavity

Consider first the in-line type, Fig. 1(a), with lengths L_A and L_B; diameters d_1, d_2, and d_3; characteristic impedances[3] Z_A and Z_B; terminating capacitance C; and fringing capacitance C_F,[4] and assume that the conductors have infinite conductivity.[5] The impedance expression $Z(f)$ for this case (given in detail by Waltz [1]) is sufficiently complicated that an easy determination of the ranges of the variables over which solutions exist is impossible. Most of the difficulty arises due to the complex inter-relationship of C_F and the cavity dimensions. Since in the applications contemplated in this paper C_F is usually not large compared to C, C_F will be neglected in the initial approach to the problem. Solution of even this simplified problem in terms of known functions is for practical purposes not possible, but solution by numerical methods is not difficult. Fig. 2 shows part of the solution to this problem in terms of the quantities r, X, θ_A, and θ_B for the case $n=2$, where

$$r = \frac{Z_B}{Z_A}, \qquad X = \frac{1}{2\pi f_1 C_0 Z_B},$$

$$\theta_A = \frac{2\pi f_1 L_A}{c}, \quad \text{and} \quad \theta_B = \frac{2\pi f_1 L_B}{c}.$$

Choice of a particular value for any one of the quantities, r, X, θ_A or θ_B immediately determines the other three. But since r and X depend on *ratios* of diameters, one degree of freedom still remains. That is, one of the three diameters can be chosen arbitrarily, after which

[3] Unless otherwise stated air dielectric is assumed.

[4] "Fringing capacitance" is a convenient device for representing the effect of the discontinuity in the center conductor. Whinnery, Jamieson, and Robbins have shown [2] [3] that the types of discontinuities considered in this paper can be accurately represented by one or more lumped frequency-independent capacitances, with the usual transmission-line equations applying to both sides of the capacitances. Further discussion of "fringing capacitances" and their effects is included elsewhere in this paper.

[5] This cavity configuration permits the independent choice of five parameters: d_1, d_2, d_3, L_A, and L_B. (C_F is not independent, but depends on the cavity dimensions [2], [3].) Thus, it may be possible to satisfy as many as five independent design conditions. A few of the many conditions which might be of interest in particular applications are as follows: 1) Synthesize for a specified Z_A, 2) Synthesize for a specified Z_B, 3) Synthesize for specified diameters, 4) Minimize overall cavity length L_A+L_B, 5) Synthesize so that the resonant frequencies shift as little as possible over a range of values of terminating capacitance. Only the two primary conditions (dual-resonance and insensitivity) are considered in detail in this paper, however.

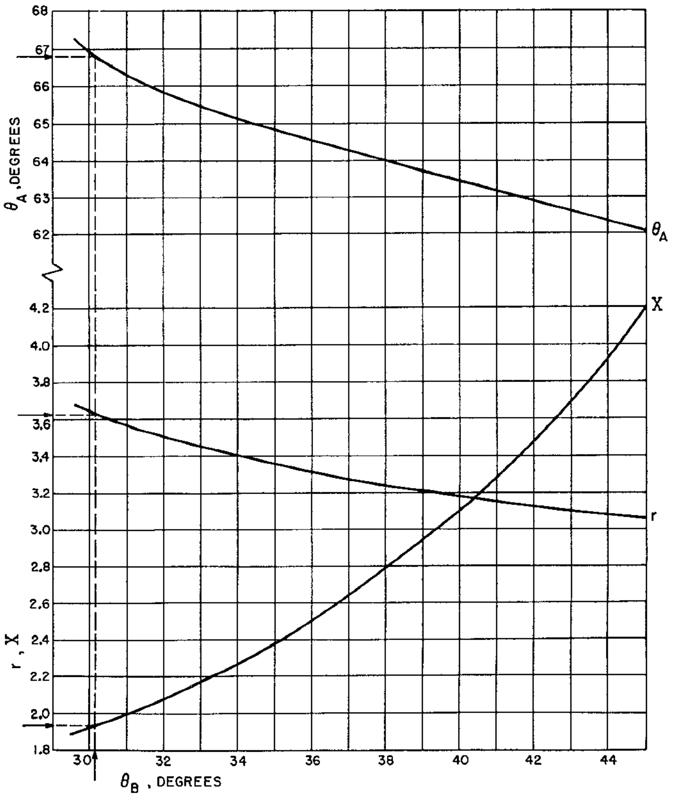


Fig. 2—Solutions for in-line cavity for $n=2$ neglecting fringing capacitance.

the other two diameters can be determined from the definitions of r and X using the known values of f_1, C_0, r, and X, and the formulas

$$\frac{d_3}{d_2} = \log_e^{-1}\frac{Z_A}{60} \tag{7}$$

and

$$\frac{d_3}{d_1} = \log_e^{-1}\frac{Z_B}{60}\,. \tag{8}$$

Effect of Fringing Capacitance

Analysis of the original model (hereafter referred to as the fringing capacitance model) is considerably more complicated than that of the simplified model. Also, because C_F depends on actual diameters, rather than ratios of diameters, solutions must be tabulated as a function of *two* parameters rather than the single parameter θ_B that sufficed for the results shown in Fig. 2.

To solve this problem, a digital computer was programmed[6] to find solutions to (5) for $n=2$ for specified values of C_0, f_1, d_1, and d_3, and for a series of values of d_2, and at the same time to calculate the value of each side of (6) for each solution, thus allowing the cavity designer to pick (by interpolation) the one value of d_2 and the corresponding values of θ_A and θ_B which also satisfy (6). Fig. 3 is a plot of the computed results for the particular case $n=2$, $f=775$ Mc, $C=1.75$ pf, $d_1=0.3414$ inch, and $d_3=0.936$ inch. Arrows indicate the required values of θ_A, θ_B, and d_2.

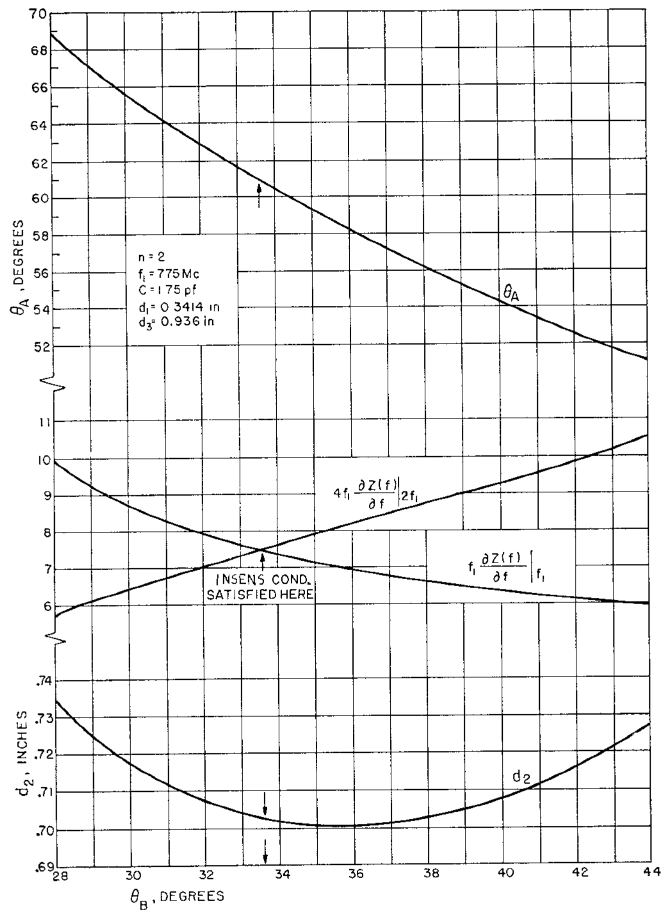


Fig. 3—Plot of typical results for in-line cavity, based on the fringing capacitance model.

To satisfy one (or two) condition(s) in addition to the dual-resonance condition and the insensitivity condition, a series of solutions for various values of d_1 and d_3 could be obtained from which the desired solutions (or solution) could be selected.

Comparison of Results for Simplified Model with Results for Complete Model

To get some estimate of the error introduced by the neglect of fringing capacitance, a comparison was made for the particular case covered by Fig. 3. Fig. 2 was used to determine the corresponding solution of the simplified problem for the same values of d_1 and d_3 (*i.e.*, $Z_B=60.5136$ ohms), and for $C_0=1.75$ pf, and $f_1=775$ Mc (which result in $X=1.939$). The arrows on Fig. 2 indicate the resultant values of θ_A, θ_B, and r. Fig. 4 shows how the two results compare. The conclusion that can

[6] The value of C_F was determined in each case from equations which were fitted to the curves given by Whinnery and Jamieson [2], [3]. If a particular application requires more accurate calculation of C_F than is possible by this method, then the complete procedure given in these references could be used.

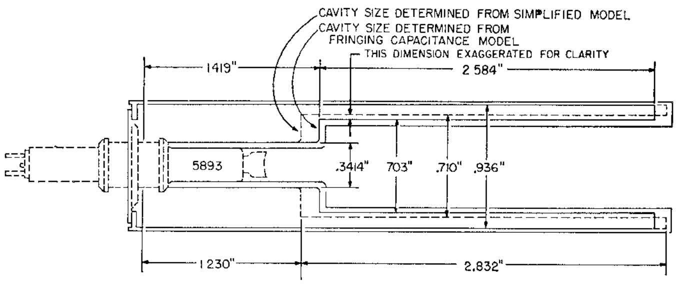


Fig. 4—Comparison of results from simplified model and fringing capacitance model.

be drawn, in this case at least, is that, while the simplified model can be used to determine rather closely the *ranges* of d_2, θ_A, and θ_B over which solutions based on the fringing capacitance model can be expected, it is not sufficiently accurate to be used as a basis for actual cavity construction.

Re-Entrant Cavities

A major portion of the over-all length of the cavity shown in Fig. 4 is taken up by the low-impedance section of coaxial line. Further study shows this to be generally true of the in-line type of cavity. The re-entrant designs shown in Fig. 1(b) and (c) make possible a considerable saving in both length and volume. Another advantage of the re-entrant designs is that, since these cavities have seven parameters (*i.e.*, d_1, d_2, d_3, d, L_A, L_B, and L_C), it may be possible to satisfy seven design conditions, as opposed to the five conditions that were possible with the in-line type. The use of a dielectric other than air in the re-entrant section[7] adds an additional parameter to the system, bringing the total to eight.

The analyses of the re-entrant configurations are orders of magnitude more difficult than that of the in-line configuration, due to the additional parameters and also due to the fact that *three* fringing capacitances, C_A, C_B, and C_C, located as shown in Fig. 1(b) and (c), must be taken into account [2], [3].

Digital computer programs were written to analyze both the external and the internal re-entrant configurations. Fig. 5 presents the results for the external re-entrant cavity for the special case $n=2$, $f_1=500$ Mc, $d_1=0.28125$ inch, $d_3=1.436$ inches, $d=0.01$ inch, and $\epsilon_r=1.00$. This figure can be applied to the design of particular cavities by choosing the capacitance that is to be resonated and a value for one other parameter (L_A, L_B, L_C, or d_2), and then reading from the figure the values for the remaining parameters. As before, additional conditions could perhaps be satisfied by other choices of d_1, d_3, d, and ϵ_r. The limited ranges of the parameters shown in Fig. 5 do not necessarily imply

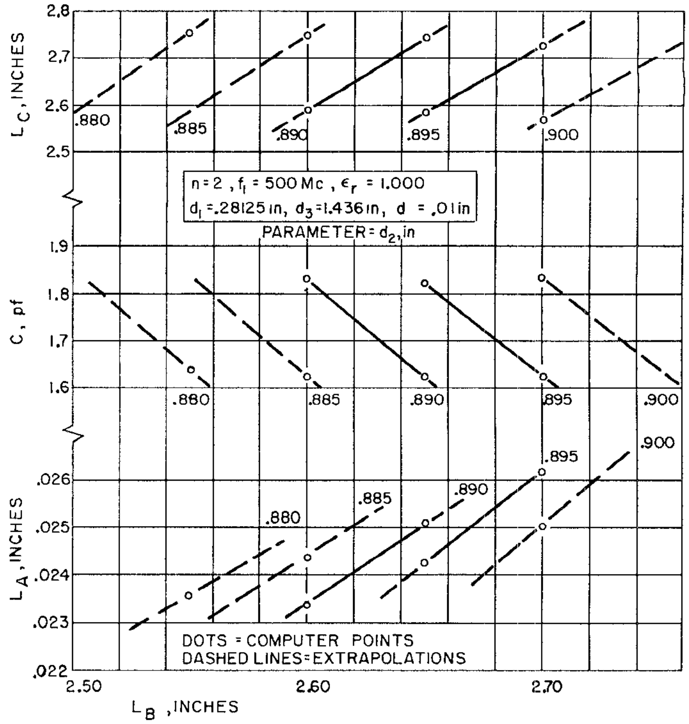


Fig. 5—Plot of typical computer results for the external re-extrant cavity model.

that no solutions exist outside of these ranges. However, in every special case that was investigated, the ranges were severely limited (particularly the range of d_2). Usually one or more exploratory computer runs had to be made before the problem could be narrowed down to the ranges in which solutions were possible.

In this connection, it should also be noted that all the examples considered in this paper involve standing-wave patterns analogous to a quarter-wave mode for the fundamental frequency and a three-quarter-wave mode for the second harmonic. Other mode combinations are possible but were not considered here because minimum over-all length was desired.

The internal re-entrant model was investigated for the special case $n=2$, $f_1=600$ Mc, $\epsilon_r=2.10$ (Teflon), $d=0.28125$ inch, $d_3=0.936$ inch, and $d=0.01$ inch. Some solutions which satisfied both conditions were found, but all of them involved either small values of C or values of L_B larger than L_C [a physically unrealizable configuration in most practical applications, as inspection of Fig. 1(c) will reveal]. Trends observed in the computer output indicated that, in this case at least, a larger value of ϵ_r might yield more practical results. Further investigation of the internal re-entrant model was not carried out, however, because it was assumed that the validity of the approach and the approximations used could be adequately tested on the basis of results for the other two cavity types.

III. Application to Cavity Design

Actual coaxial cavities typically depart from the models described above in two important respects:

[7] If, as in the in-line case, the low-impedance section turned out to be longer than the high-impedance section, the internal re-entrant configuration could not be used in many cases, since the center conductor would not be long enough to accommodate the desired length of re-entrant line. A dielectric allows the *effective* length of the re-entrant line to be increased.

1) some losses are present, and
2) the simple open-end construction assumed in the analyses is usually not used.

Losses are present due to the finite conductivity of the conductors, the external loading, and the shunt conductance of the terminating device (vacuum tube "varactor," etc.). When these losses are reasonably small, however, they principally affect the Q of the cavity and have only minor effect on the resonant frequencies. Thus the models used here can be expected to give a reasonable approximation to the performance of well-designed and well-constructed cavities in many applications.

The conditions at the open end of actual cavities can also be fitted to the model, in many cases. A typical UHF vacuum tube, such as the one shown in Fig. 4, has anything *but* a constant conductor diameter inside the tube. Pencil triodes even have a re-entrant section inside the tube, since the grid is coaxial with the hollow plate conductor. It has been shown by Whinnery, Jamieson and Robbins [2], [3], however, that, if the cavity outside diameter is large compared to these internal conductor diameters of the tube, the fringing capacitances are small. Furthermore, the many individual fringing capacitances typically present (due to the many changes in diameter and dielectric constant in and near the tube) are typically located within a region having dimensions small compared to a wavelength, so that they can often be well approximated by a single fringing capacitance at a properly chosen location.[8] Note also that, because of the satisfaction of the insensitivity condition, the operation of the cavity will be insensitive to any small variations with frequency of the equivalent capacitance. Thus, it appears that the models considered here may be useful for the design of actual coaxial cavities.

IV. Experimental Results

To test the validity of the previously mentioned analyses, experimental cavities were constructed and tested. The lengths of the cavities were made adjustable, so that dimensions *near* as well as *at* the design dimensions could be evaluated.

Fig. 6 shows the experimental results for an in-line cavity with $d_1 = 0.3414$ inch, $d_2 = 0.703$ inch, and $d_3 = 0.936$ inch, which is the case discussed in Section II and shown in Fig. 4 (solid lines). Fig. 6(a) reveals that the desired result can indeed be achieved, and at dimensions very close to the calculated dimensions. The pairs of curves in Fig. 6(b) show the effects of variations in the length dimensions away from the optimum values. It is

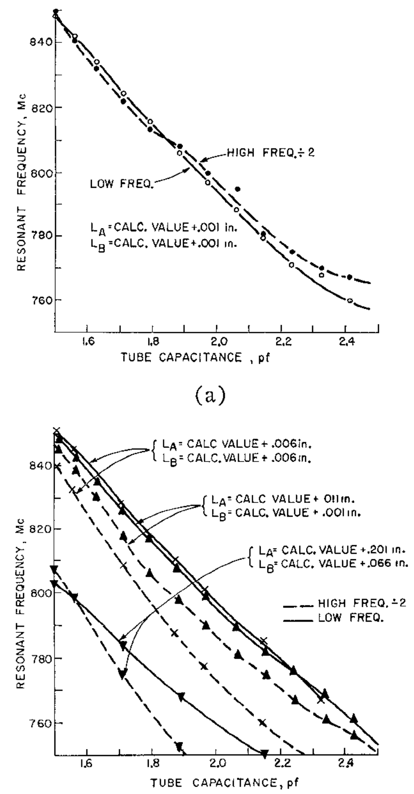


Fig. 6—Experimental results for in-line cavity. (a) Dimensions essentially equal to the calculated values. (b) Dimensions deviating from the calculated values.

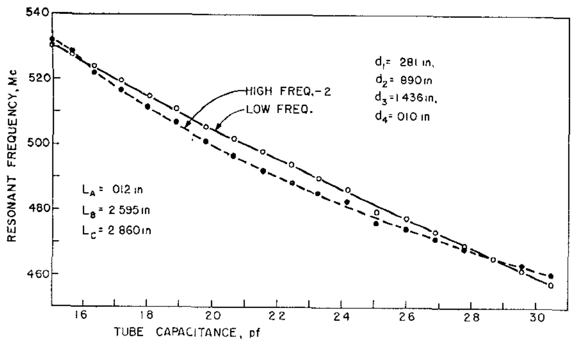
apparent from these curves that, for small variations at least, the low-frequency curves are less sensitive to dimensional changes than the high-frequency curves, and that changes in L_B are more critical than changes in L_A. The lowest pair of curves is included to show that dimensions which deviate significantly from the calculated values produce greatly inferior results, and hence that the obtaining of the desired behavior is indeed neither a coincidence nor an inherent property of all such cavities. This implies, further, that the design of such cavities by purely experimental means would be very time consuming.

The experimental external re-entrant cavity model was built to conform as closely as was practical[9] to the special case covered by Fig. 5. Lengths L_A, L_B, and L_C [see Fig. 1(b)] were adjustable. In this case, because there are three variables and only two conditions to be satisfied, a family of calculated results, rather than a single result, is to be expected. Therefore, exploration of a region including part of the calculated family, rather than verification of a single calculated design, was undertaken.

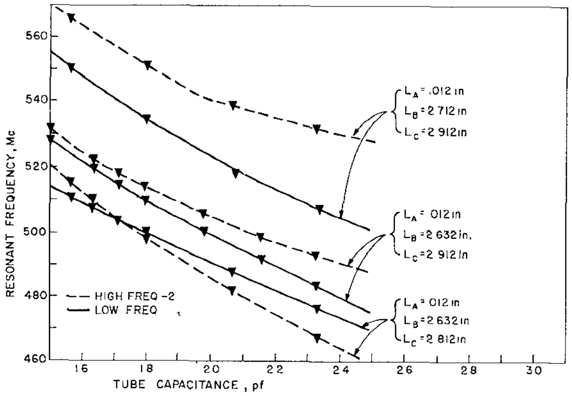
Fig. 7 shows some of the results obtained. Note that, while the separation between the curves in Fig. 7(a) is in general greater than that shown in Fig. 6(a), the curves

[8] The equivalent capacitance can be determined with the method of Whinnery and Jamieson, [2], [3], or by direct experiment. Experiments performed with a 5893 pencil triode in a simple cavity (1 inch outer conductor, 5/16 inch inner conductor) gave a value of C which was constant within 5 per cent over the range investigated, 600 to 1400 Mc. Corresponding results can be expected for other devices and frequencies, as long as the wavelength is not too short.

[9] A 0.010-inch-thick copper sheet was rolled and soldered into a tube, which was used as the re-entrant wall. This construction technique resulted in some variation in diameter and in concentricity.



(a)



(b)

Fig. 7—Experimental results for external re-entrant cavity.

in Fig. 7(a) remain close together over a wider range of capacitance variation. Limitations in the experimental equipment prevented checking the performance over an even wider capacitance range, but it appears from Fig. 7(a) that the curves should remain close to each other over a capacitance range considerably beyond two-to-one. Alternatively, some of the capacitance range could be sacrificed for the sake of better midrange performance, if the high-frequency curve could be shifted upwards slightly. There is good reason to believe that a set of values for L_A, L_B, and L_C near those used in obtaining Fig. 7(a) would accomplish this result. Fig. 7(b) shows that here, as in the case of the in-line configuration, good performance is not a foregone conclusion, and hence that the calculated results eliminate the need for a great deal of exploratory experimentation.

In general, the experimental results for the re-entrant cavity did not agree with the calculated results as well as in the in-line case previously considered.[10] Nonetheless, the analyses for both configurations yielded results very close to those determined experimentally. For optimum performance in a given application, some tuning may be needed, but the analyses allow a drastic reduction in the range of the variables over which a solution must be sought. In the case of the re-entrant design, this is especially important, since an experimental cavity involving eight or nine manipulated variables (even if it could somehow be built) would be almost impossible to investigate experimentally in a reasonable length of time.

[10] This is believed to be due in part to the fact that the fringing capacitance approximations used become somewhat less accurate at the small values of L_A used, and in part to the limitations of the experimental setup, which rendered impractical an exact embodiment of the mathematical model and a full investigation of all combinations of L_A, L_B, and L_C which might have been of interest.

V. Related Problems

The results of this investigation, and the success of the mathematical model in dealing with this problem, suggest that the same techniques might be effective in the solution of different but related problems, some of which are as follows:

Synthesis of cavities which resonate at three or more frequencies which are in specified ratios, harmonically-related or not.

Synthesis of cavities which resonate at two or more frequencies which maintain constant frequency separations (as opposed to frequency ratios) over a range of terminating capacitances (*i.e.*, for mixer applications).

Control of antiresonant frequencies in addition to or instead of resonant frequencies (*i.e.*, for filter applications).

Synthesis of tuning or modulation devices of special characteristics, using voltage-variable capacitors to obtain capacitance variation.

VI. Conclusions

This paper has considered primarily coaxial cavities which satisfy both the dual-resonance condition and the insensitivity condition. The mathematical model which has been used predicts that the desired conditions can be met. Experimental cavities built according to computed results gave performance very close to the predicted performance, thus proving that the desired results are indeed obtainable, and demonstrating the usefulness of the method of analysis. Therefore, it should be possible, with a minimum of experimentation and/or tuning, to design and build coaxial cavities which resonate at two desired frequencies (harmonically-related or not), in spite of wide variations in the terminating capacitance.

Appendix I

Dual Resonance in Uniform Coaxial Cavities

Dual Resonance Condition

Eq. (1), which pertains to uniform coaxial cavities, has any number of solutions which satisfy the dual-resonance condition. None of these solutions meets the conditions required in this paper, for reasons described below.

For this simple type of cavity, the appropriate form of the impedance expression is

$$Z(f) = jZ_0 \tan \frac{2\pi f L}{c} \cdot$$

Applying the dual-resonance condition (5) to this expression, defining

$$\theta = \frac{2\pi f_1 L}{c},$$

and setting $f_2 = nf_1$, gives, after simplification,

$$\tan\theta = n \tan n\theta \quad \text{(dual-resonance condition)}. \tag{9}$$

The two sides of this equation are plotted in Fig. 8 for the case $n=4$. There are four solutions to (9) in the interval $0 \le \theta \le \pi$: $\theta = 0$, 0.8554, 2.4862, and π. Corresponding values appear in each interval $k\pi \le \theta \le (k+1)\pi$, $k = 1, 2, \cdots$.

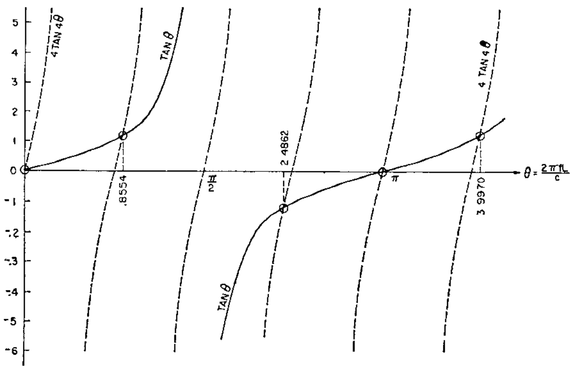


Fig. 8—Example of the satisfaction of the dual-resonance condition by a uniform coaxial cavity. $n=4$.

The cases $\theta = 0, \pi, 2\pi, \cdots$ correspond to infinite values[11] of C (or Z_0), and hence are of practical value in this application only when C or Z_0 are so very large that they approximate this case. The case $\theta = 0.8554$ corresponds to $f_1CZ_0 = 0.1382$, which is a meaningful solution satisfying the dual-resonance condition. Where sensitivity to variations in terminating capacitance is not important, this solution could be used. It is shown below, however, that this solution (as well as all others possible with a uniform cavity) does not satisfy the insensitivity condition. The case $\theta = 2.4862$ corresponds to $f_1CZ_0 = -0.1382$, which requires negative values of C.

If figures corresponding to Fig. 8 are plotted for $n = 2, 3, 5, 6$, etc., it becomes apparent that only one other type of solution is possible. Solutions of this type occur at

$$\theta = \frac{\pi}{2}, \quad \frac{3\pi}{2}, \quad \frac{5\pi}{2}, \cdots,$$

and involve $\tan\theta = n \tan n\theta = \infty$, which corresponds to $C = 0$. These are not useful solutions to the problem posed in this paper, for reasons analogous to those given for the case $C = \infty$.

Insensitivity Condition

Applying the insensitivity condition (6) to

$$Z(f) = jZ_0 \tan \frac{2\pi f L}{c}$$

gives

$$jZ_0 \cdot \frac{2\pi L}{c} \sec^2 \frac{2\pi f_1 L}{c} = n^2 \left(jZ_0 \frac{2\pi L}{c} \sec^2 \frac{2\pi n f_1 L}{c} \right) \quad \text{(insensitivity condition)}, \tag{10}$$

which, upon simplification and use of the identity $\sec^2\theta = 1 + \tan^2\theta$, reduces to

$$1 + \tan^2\theta = n^2 + n^2 \tan^2 n\theta \quad \text{(insensitivity condition)}. \tag{11}$$

Since, by assumption, the dual-resonance condition (9) is satisfied, this further reduces to

$$1 = n^2 \quad \text{(insensitivity condition)}. \tag{12}$$

But this is the trivial case $n = 1$ which is not even a dual-resonance solution in the sense used here. Hence, it is impossible for a uniform coaxial cavity to satisfy both the dual-resonance condition and the insensitivity condition.

[11] At a particular frequency a capacitor can be made to approximate an infinite capacitor by means of a properly-adjusted single stub tuner—*i.e.*, the capacitor can be "resonated" at that frequency. This combination will not be resonant at the harmonic frequency, however, and can not be used to meet the dual-resonance condition as described here. Multiple-stub tuners might accomplish the desired results, at the cost of greater complexity and greater difficulty in analysis. Realizing that stub tuners are nothing more than cavities themselves, one perceives that these cavity-stub tuner circuits replace the radially-symmetric series or series-parallel cavity combinations shown in Fig. 1 with parallel or series-parallel circuits of a type lacking radial symmetry. It is this lack of radial symmetry which causes the increased analytical difficulties which are associated with the stub-tuner circuits. (Simple models, neglecting fringing capacitance, can not be expected to suffice for these circuits any more than for the multiple-section cavities considered in this paper.)

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